

# Data Assimilation in Ocean Prediction

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## LONG-TERM GOAL

The long-term goal of this project is to develop computationally efficient and optimal techniques for the direct assimilation of satellite altimetry data into the global and mesoscale ocean circulation models in order to accurately reconstruct the ocean circulation fields thereby improving the capability of ocean nowcast and forecast.

## OBJECTIVES

The primary objective of our work is to interface fast implementations of the Kalman-Bucy filter (KBf) that we have developed [1, 2, 3, 4, 5] with the Naval Research Laboratory (NRL) Layered Ocean Model (NLOM) and then carry out extensive data assimilation studies. We are working in close collaboration with the group of Dr. Harley Hurlburt at the NRL Stennis Space Center.

## APPROACH

We assimilate altimetry data into NLOM by decoupling the ocean circulation fields into deterministic and random components. We apply dynamic linearization. Dynamic linearization involves two stages at each time step: the nonlinear deterministic stage, and the linear stochastic stage. With the nonlinear deterministic stage, the ***nonlinear*** stochastic primitive equations is integrated forward during the current time step from assumed deterministic known boundary conditions, known initial conditions, and known driving forcings. This first nonlinear stage uses the full nonlinear set of primitive equations but considers only the nominal operating conditions: boundary conditions, initial conditions, and forcings. The second linear stochastic stage of dynamic linearization linearizes the nonlinear primitive equations about the deterministic nonlinear solution obtained from the first stage. The nonlinear deterministic equations in the nonlinear stage of dynamic linearization are obtained with the NLOM. The linearization of the primitive equations in NLOM gives the state equation for the KBf. The measurement equation for the KBf is provided by the satellite altimetry scan pattern. The KBf can then use the linearized equations and the satellite measurements to estimate the random components of the circulation fields. Once these estimates are available, they are used to correct at the current time step the nonlinear deterministic field components. These corrected nonlinear deterministic components provide then the initial and boundary conditions for the nonlinear stage of dynamic linearization. The process repeats at every time step. When no new measurements are available, the dynamic linearization keeps marching forward through the nonlinear primitive equations.

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Direct implementation of the KBf leads to a computationally unfeasible problem. To overcome this we exploit the block structure of the model. In particular, the system matrices associated with the linearized set of primitive equations and the KBf exhibit a banded-block structure that enables us to reduce significantly the computational cost of the KBf. The local nature resulting from the discretization of the state and measurement equations of the KBf is naturally reflected in the block-banded structure of the system matrices, with blocks themselves being banded and sparse. By coupling this block structure with the sparsity of the satellite measurements, we obtain computationally **efficient** and **exact** implementations of the KBf, reducing its computational complexity by a factor equal to the linear dimension  $I$  of the discretized grid.

To reduce further the computational complexity, we approximate the information matrix of the KBf, i.e., the inverse of the error covariance matrix, by a sparse banded matrix. Banded approximations to information matrices correspond to modeling the error field in our spatial estimates at each point in time as a reduced order Markov random field (Mrf). It can be shown that these banded inverse approximations are optimal approximations in a maximum entropy sense. We apply the block-banded structure to derive several recursive algorithms, which relate the constituent blocks in the error covariance matrix to its diagonal and its upper diagonal entries. Due to the special sparse nature of the measurements, most of the off diagonal blocks need not be updated. We update only the main and the upper diagonal blocks. Any other block if needed may be derived from the Mrf relationships. This reduces the computational effort for the single layer NLOM by at least  $O(I^2)$  over the direct KBf. The storage requirements are reduced by  $O(I)$ . Our implementations provide additional efficiencies with multiple layers.

## WORK COMPLETED

Assimilation of the altimetry data has been investigated for the latest spherical version of the NLOM. The derivation of the data assimilation algorithm based on the KBf was completed and fine-tuned for improved performance. We ran experiments with the NLOM at a finer resolution of  $\frac{1}{4}$  degree for a more complex three and a half hydrodynamic NLOM in several regions of interest. Experiments were initially run using both analytical winds and the more realistic annually averaged Rosenthal winds.

In the near future, extension to real Topex / Poseidon data is planned. This will make our experimentation more realistic.

## RESULTS

We provide results for a  $\frac{1}{4}$  degree three and a half layer hydrodynamic NLOM lying within  $-10^\circ < \text{latitude} < 10^\circ$ ,  $306^\circ < \text{longitude} < 374^\circ$ . Figure 1 shows contour plots of the estimates of the sea surface height (SSH) for the top layer after 5 Topex/ Poseidon simulated repeat cycles. The top plot in Figure 1 is the “truth”, the middle plot in figure 1 is the spin up with no assimilation, and the bottom plot in figure 1 is the field after data assimilation. Close visual comparisons of the plots reveal that there are several major differences in the three plots. The higher resolution details like eddies are completely absent from the middle plot of figure 1 while most of them appear at the correct position in the assimilated plot at the bottom of figure 1.

In figure 2, we plot for the top two layers the time evolution of the relative mean square error (RMSE). The RMSE is defined as the ratio of the MSE of the data assimilated run over the MSE of the deterministic run. With time, the RMSEs for both layers drop suggesting an improvement in the data assimilated prediction. The improvement is higher, roughly 25% at the end of the experiment, for the upper layer for which the satellite provides direct measurement data. No data is assimilated for layer 2, yet we see an overall improvement of about 10%. Our KBF implementation successfully correlates the two layers and projects the surface information into the lower layers.

## IMPACT/APPLICATION

We have presented our work at the Third World Meteorological Organization (WMO) International Symposium on Assimilation of Observations in Meteorology and Oceanography, Quebec City, Canada, see reference [1] below. We obtained good and encouraging reviews from the participants.

A full paper on this work was also accepted and will appear in the November 1999 issue of the IEEE Transactions on Image Processing, see reference [2].

The project is in close collaboration with the research group of Dr. Hurlburt at the NRL Stennis Space Center. Our goal is to make available the results to NRL. This work will lead to improved ocean forecasts that can be used with the Navy fleet nowcast and forecast programs.

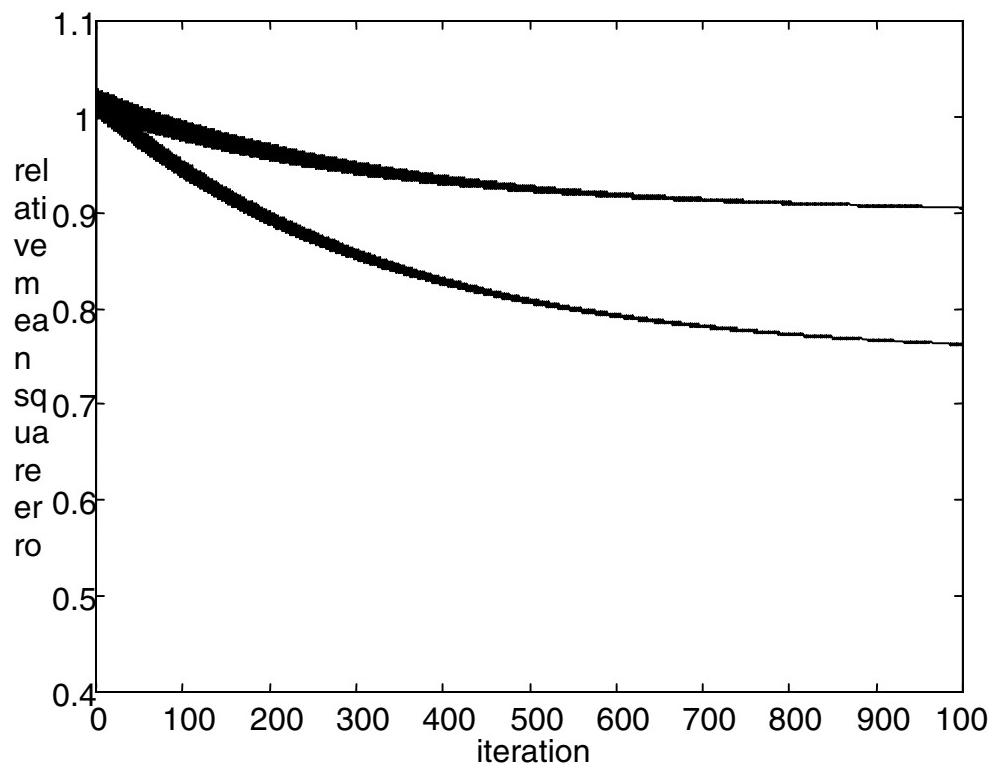
## TRANSITIONS

The code will be made available to NRL.

## REFERENCES AND PUBLICATIONS

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*1: Contour plots of the sea surface height (SSH): Top: SSH for the actual truth; middle: SSH with no data assimilation; and bottom: SSH with data assimilation from satellite altimetry. Darker regions are deeper, lighter regions are higher.*



2: RMSE of the SSH for the top two layers. The lower curve is the RMSE for the surface layer while the upper curve is the RMSE for the second lower layer. No data is assimilated for the lower layers, only altimetry data for the top layer is assimilated yet reconstruction of all layers is significantly improved.